Review : Integrating Clipped Spherical Harmonics Expansions

- Spherical Harmonics expansion
  - Any spherical function can be decomposed in an infinite SH expansion
  - Spherical Harmonics are an orthonormal basis of functions defined on the unit sphere
- Power cosine integration
  - Recursive integration
  - Store previous term and compute next power cosine
- SH approximates any function on sphere

Refractive Radiative Transfer Equation

MARCO AMENT, CHRISTOPH BERGMANN, and DANIEL WEISKOPF

Presentor : Mingi Lim

2018. 11.22

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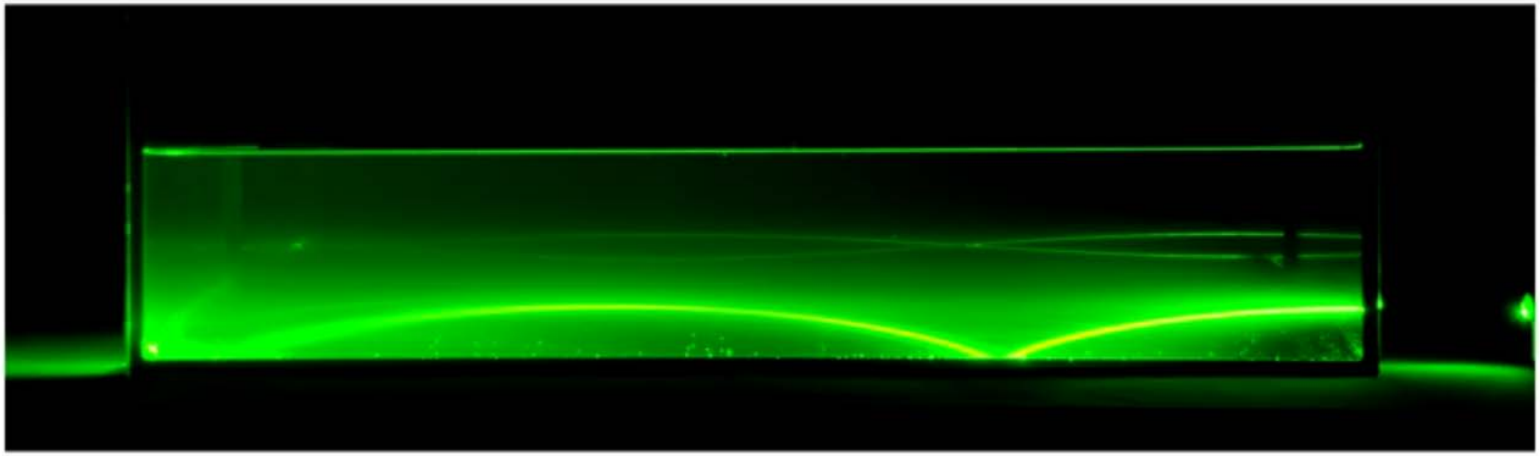
Continuous Refraction in Real World

Mirage



Sugar solution

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Overview

- Fundamental part of physics
  - [Chandrasekhar 1950, 1960] introduces refractive radiative transfer equation **w/o counting for refractive media**
  - [Pomraning 1973, 2005] introduces **radiance divided by square of radiative index** as a conservative quantity -> strongly simplified equations

$$\mathcal{L} = \frac{L}{n^2}$$

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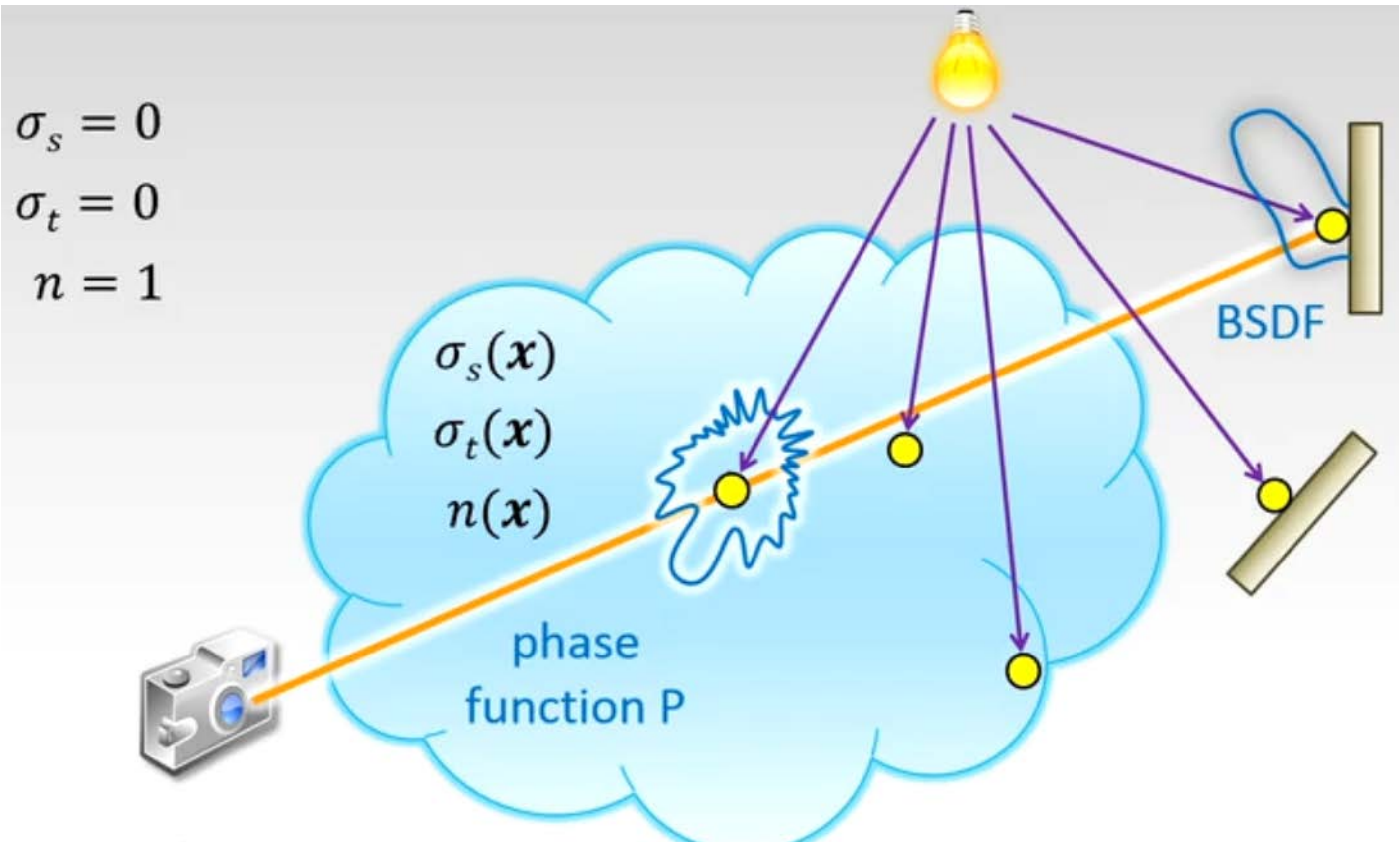
Overview



$$\frac{dL}{ds} = -\sigma_t L + \frac{\sigma_s}{4\pi} \int_{\Omega} P(\boldsymbol{\omega}', \boldsymbol{\omega})\, L(\boldsymbol{x}, \boldsymbol{\omega}')\, d\omega'$$

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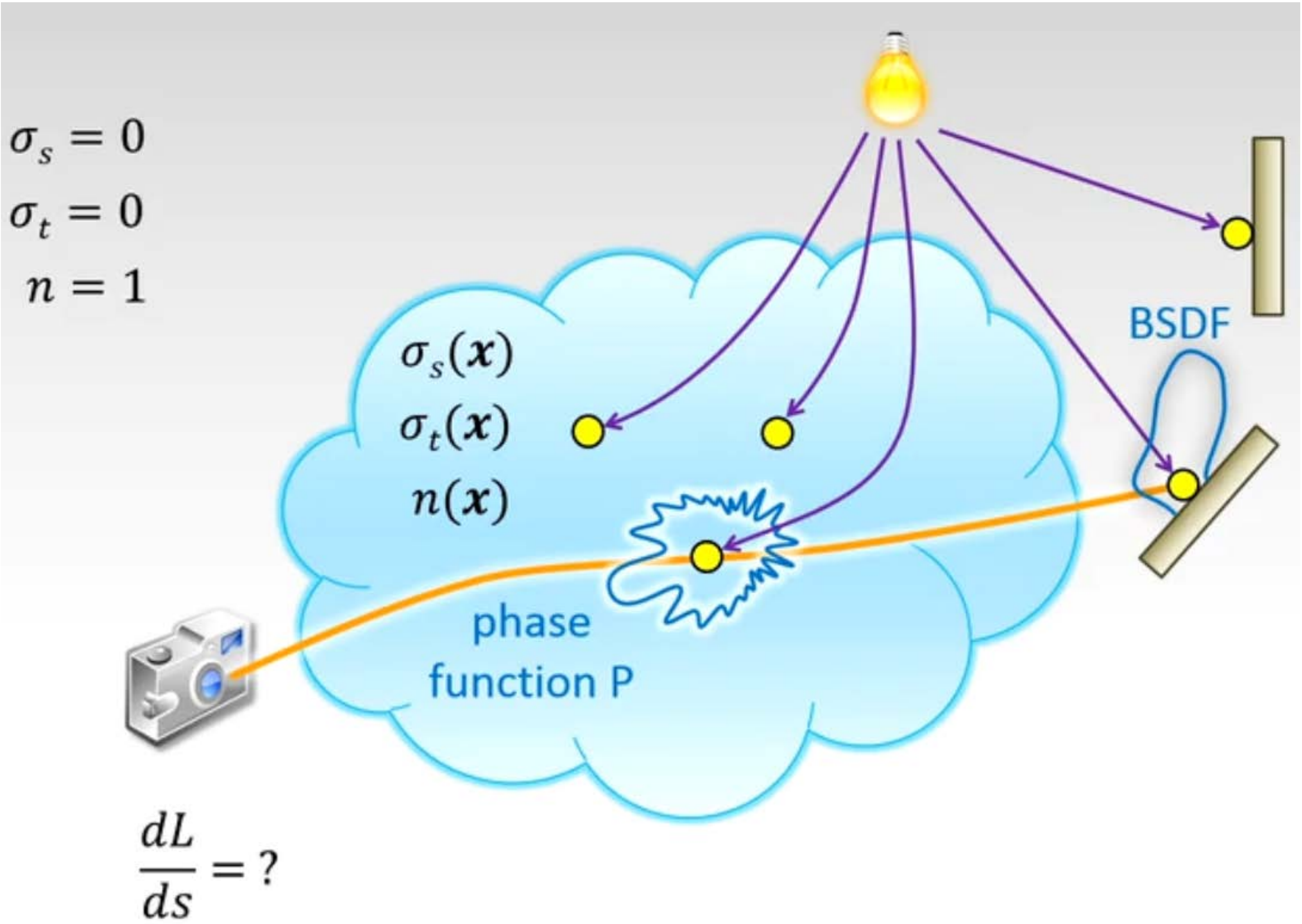
Overview



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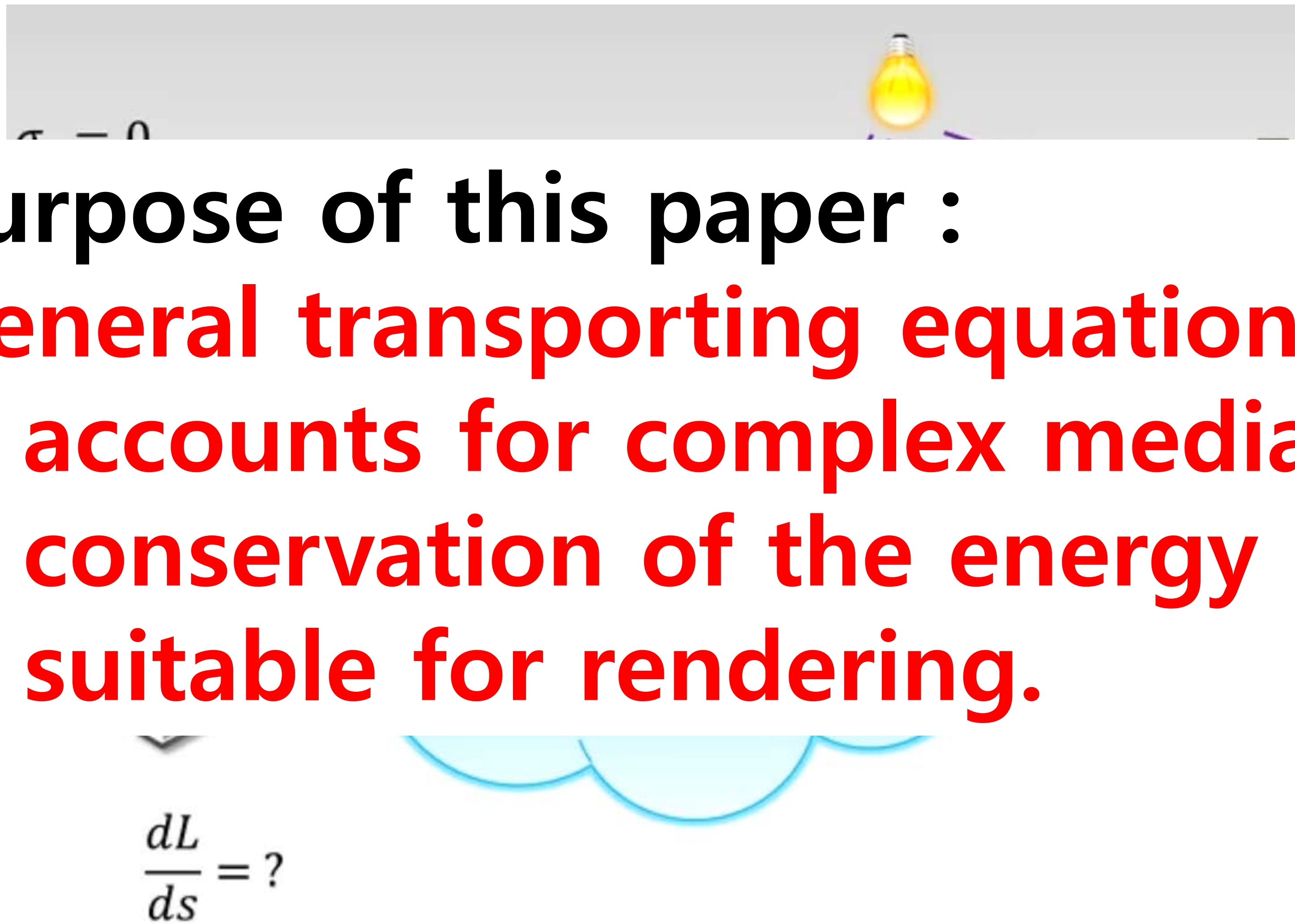
Overview



Purpose of this paper :

General transporting equation

- **accounts for complex media**
- **conservation of the energy**
- **suitable for rendering.**

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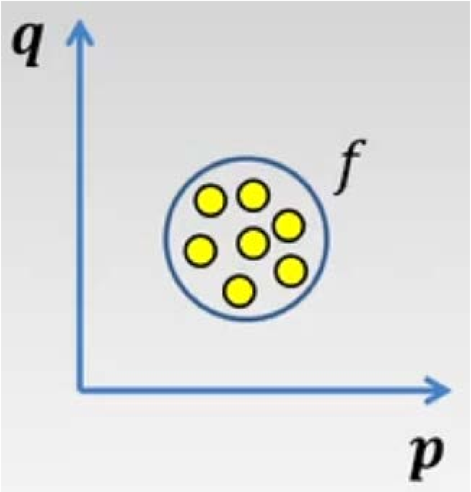
Backgrounds

Physics Background

- Hamiltonian mechanics
  - Developed as a reformulation of classical mechanics and predicts the same outcomes as non-Hamiltonian classical mechanics.
  - Classical mechanical system ->
    a phase space composed of **coordinates** and **corresponding momentum**

Physics Background

- Photons in 6D phase space
  - Position $\boldsymbol{p} = (p_1, p_2, p_3)$
  - Momentum $\boldsymbol{q} = (q_1, q_2, q_3)$
  - Density $f(\boldsymbol{p}, \boldsymbol{q}, t)$



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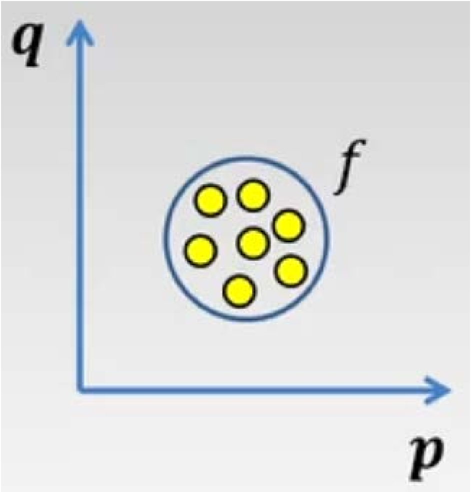
  - Density $f(\boldsymbol{p}, \boldsymbol{q}, t)$
- Liouville's Theorem [Liouville 1838]

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(\frac{\partial (f \dot{p}_i)}{\partial p_i} + \frac{\partial (f \dot{q}_i)}{\partial q_i} \right) = 0$$



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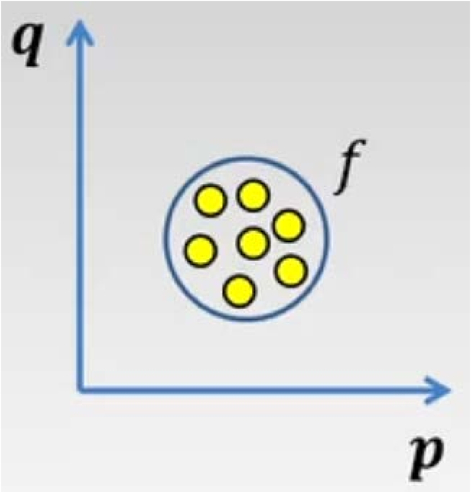
- Substitute canonical coordinates

$$\boldsymbol{p} = (p_1, p_2, p_3) \rightarrow (x, y, z) = \boldsymbol{x}$$

$$\boldsymbol{q} = (q_1, q_2, q_3) \rightarrow (\emptyset, \mu, v) = (\boldsymbol{w}, v)$$



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Light Transport in Phase Space

- Participating media with no self-emission
  - Considering only **absorption** and **scattering**

- Light transport in phase space

$$Q(f) = \frac{\partial f}{\partial t} + \frac{\partial (f\dot{x})}{\partial x} + \frac{\partial (f\dot{y})}{\partial y} + \frac{\partial (f\dot{z})}{\partial z} + \frac{\partial (f\dot{\emptyset})}{\partial \emptyset} + \frac{\partial (f\dot{\mu})}{\partial \mu} + \frac{\partial (f\dot{v})}{\partial v}$$

-> Q : phase space density term, $\frac{df}{dt}$

-> later we substitute **this f term** for **radiance** for rendering compatibility.

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Light Transport in Phase Space

- Participating media(no self-emission)
  - Absorption $Q_t(f) = \left.\frac{df}{dt}\right| = -\sigma_t v_g f$
  - Scattering $Q_s(f) = \left.\frac{df}{dt}\right| = \frac{\sigma_s}{4\pi}\int_{\Omega} P(\boldsymbol{\omega}', \boldsymbol{\omega})\, v_g f \, d\omega'$
  - Combination $Q(f) = Q_t(f) + Q_s(f)$
- Relationship between radiance and density function[Pomraning 2005]

$$L(x, w, v, t) = v_g h v f(\boldsymbol{x}, \mathbf{w}, v, t)$$

Hamiltonian Optics

- Geometric optics [Born and Wolf 1999]
  - Wave length << size of scene objects
  - Superposition of discrete wave packets (photons)
  - Fermat's principle

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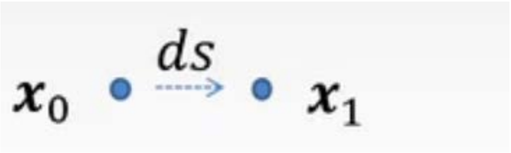
$$\frac{d\boldsymbol{x}}{ds} = \boldsymbol{w}$$



Position

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Hamiltonian Optics

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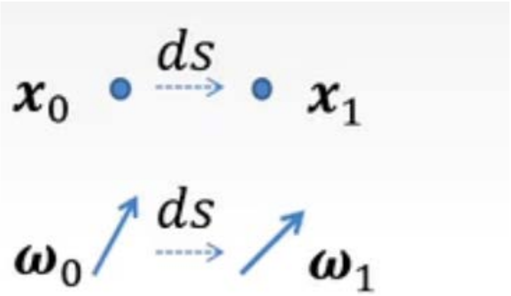
  - Wave length << size of scene objects
  - Superposition of discrete wave packets (photons)
  - Fermat's principle

$$\frac{d\boldsymbol{x}}{d\boldsymbol{s}} = \boldsymbol{w}$$

$$\frac{d\boldsymbol{w}}{d\boldsymbol{s}} = \frac{\boldsymbol{1}}{\boldsymbol{n}}(\boldsymbol{\nabla}_x \boldsymbol{n} - (\boldsymbol{w} \cdot \boldsymbol{\nabla}_x \boldsymbol{n})\boldsymbol{w})$$



Position

Direction

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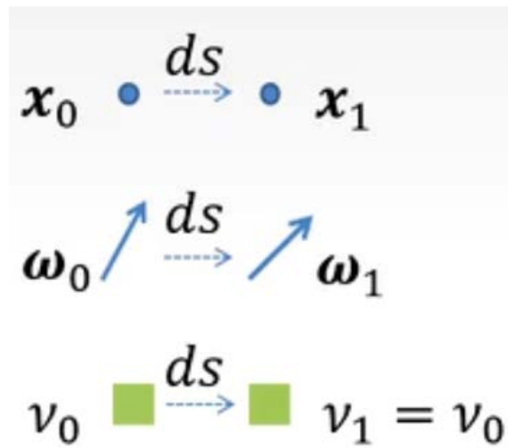
$$\frac{d\boldsymbol{x}}{d\boldsymbol{s}} = \boldsymbol{w}$$

$$\frac{d\boldsymbol{w}}{d\boldsymbol{s}} = \frac{1}{\boldsymbol{n}}\left(\nabla_x \boldsymbol{n} - (\boldsymbol{w} \cdot \nabla_x \boldsymbol{n})\boldsymbol{w}\right)$$

$$\frac{d\boldsymbol{v}}{d\boldsymbol{s}} = -\frac{\boldsymbol{v}}{\boldsymbol{c}}\frac{\partial \boldsymbol{n}}{\partial \boldsymbol{t}} = \boldsymbol{0}$$



Position

Direction

Frequency

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Hamiltonian Optics

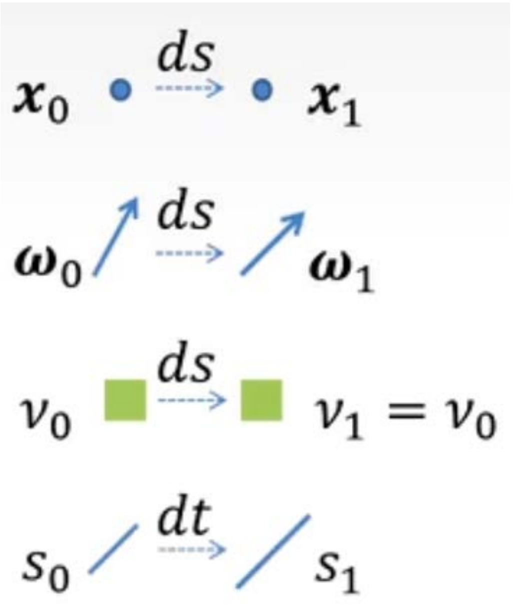
- Geometric optics [Born and Wolf 1999]
  - Wave length << size of scene objects
  - Superposition of discrete wave packets (photons)
  - Fermat's principle

$$\frac{d\boldsymbol{x}}{ds} = \boldsymbol{w}$$

$$\frac{d\boldsymbol{w}}{ds} = \frac{1}{n}\left(\nabla_x n - (\boldsymbol{w}\cdot\nabla_x n)\boldsymbol{w}\right)$$

$$\frac{dv}{ds} = -\frac{v}{c}\frac{\partial n}{\partial t} = 0$$

$$\frac{ds}{dt} = \frac{c}{n+v\frac{\partial n}{\partial v}} = v_g$$



Position

Direction

Frequency

Distance (velocity)

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Refractive Radiative Transfer Equation

Refractive RTE (RRTE)

- Important steps
  - Start with light transport in phase space
  - Substitute **density f** with **radiance L**
  - Exploit differential equations for light propagation
  - Aim toward relationships where $\tilde{L} = \frac{L}{n^2}$ is a fundamental quantity

do complex calculations …

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Refractive RTE (RRTE)

- Important steps
  - Start with light transport in phase space
  - Substitute **density f** with **radiance L**
  - Exploit differential equations for light propagation
  - Aim toward relationships where $\tilde{L} = \frac{L}{n^2}$ is a fundamental quantity

$$\frac{d\tilde{L}}{ds} = \frac{dt}{ds}\frac{\partial\tilde{L}}{\partial t} + \frac{dx}{ds}\cdot\frac{\partial\tilde{L}}{\partial x} + \frac{d\omega}{ds}\cdot\frac{\partial\tilde{L}}{\partial w}$$

$$= -\sigma_t\tilde{L} + \frac{\sigma_s}{4\pi}\int_\Omega P(\omega',\omega)\,\tilde{L}(x,\omega',\mathrm{t})\mathrm{d}\omega'$$

$\tilde{L}(x,w,t) = \frac{L(x,w,t)}{n(x)^2}$ -> Basic radiance

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Difference between RTE and RRTE

- Basic radiance
  - $\tilde{L}(x, w, t) = \frac{L(x,w,t)}{n(x)^2}$
  - The most noticeable difference between RTE and RRTE
  - Basic radiance remains constant when a ray crosses a material boundary
- Total derivative
  - $\underbrace{\frac{d\tilde{L}}{ds}}_{\text{total}} = \underbrace{\frac{dt}{ds}\frac{\partial\tilde{L}}{\partial t}}_{\text{temporal}} + \underbrace{\frac{dx}{ds} \cdot \frac{\partial\tilde{L}}{\partial x}}_{\text{directional}} + \underbrace{\frac{d\omega}{ds} \cdot \frac{\partial\tilde{L}}{\partial w}}_{\text{angular}}$

Difference between RTE and RRTE

- Continue Refraction
  - Significant advantage of RRTE
  - Comprehensive description of continuous refraction.

Solution for Steady-State RRTE

- Steady-State RRTE
  - First, define a function x(s)
    - yields the position on a curved ray
  - Second, we require exponential factor $\tau(x_0, x_1)$ [Arvo 1993] in terms of transmittance
  - Third, define a basic source term, $\tilde{J}(x, w)$
    - describes how much basic radiance is emitted or in-scattered at point x in direction w

Solution for Steady-State RRTE

- Steady-State RRTE

$$\frac{d\tilde{L}}{ds} + \sigma_t \tilde{L} = \sigma_t \tilde{J}.$$

$$\tilde{L}(\boldsymbol{x}, \boldsymbol{\omega}) = \tau(\boldsymbol{x}_0, \boldsymbol{x}) \tilde{L}(\boldsymbol{x}_0, \boldsymbol{\omega}'(\boldsymbol{x}_0)) + \int_{\boldsymbol{x}_0}^{\boldsymbol{x}} \tau(\boldsymbol{u}, \boldsymbol{x}) \sigma_t(\boldsymbol{u}) \tilde{J}(\boldsymbol{u}, \boldsymbol{\omega}'(\boldsymbol{u})) \, d\boldsymbol{u},$$

Rendering

- Basic radiance along curved beam from RRTE

$$\tilde{L}_b(\boldsymbol{x},\boldsymbol{\omega}) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}} \tau(\boldsymbol{u},\boldsymbol{x})\,\sigma_t(\boldsymbol{u})\,\tilde{L}_i(\boldsymbol{u},\boldsymbol{\omega}'(\boldsymbol{u}))\,du$$

- Solution with non-linear photon mapping
  - Trace photons with curved trajectories
  - Gather (basic) radiance with curved ray tracing
- Radiance estimation

$$L_b(\boldsymbol{x},\boldsymbol{\omega}) \approx \boxed{n(\boldsymbol{x})^2} \sum_{j=1}^{k} \tau(\boldsymbol{u}_j,\boldsymbol{x}) \frac{\sigma_t(\boldsymbol{u}_j)}{\boxed{n(\boldsymbol{u}_j)^2}} K(r_j,d_j)\,\Delta\Phi_j\,P(\boldsymbol{\omega}_j,\boldsymbol{\omega}'(\boldsymbol{u}_j))$$

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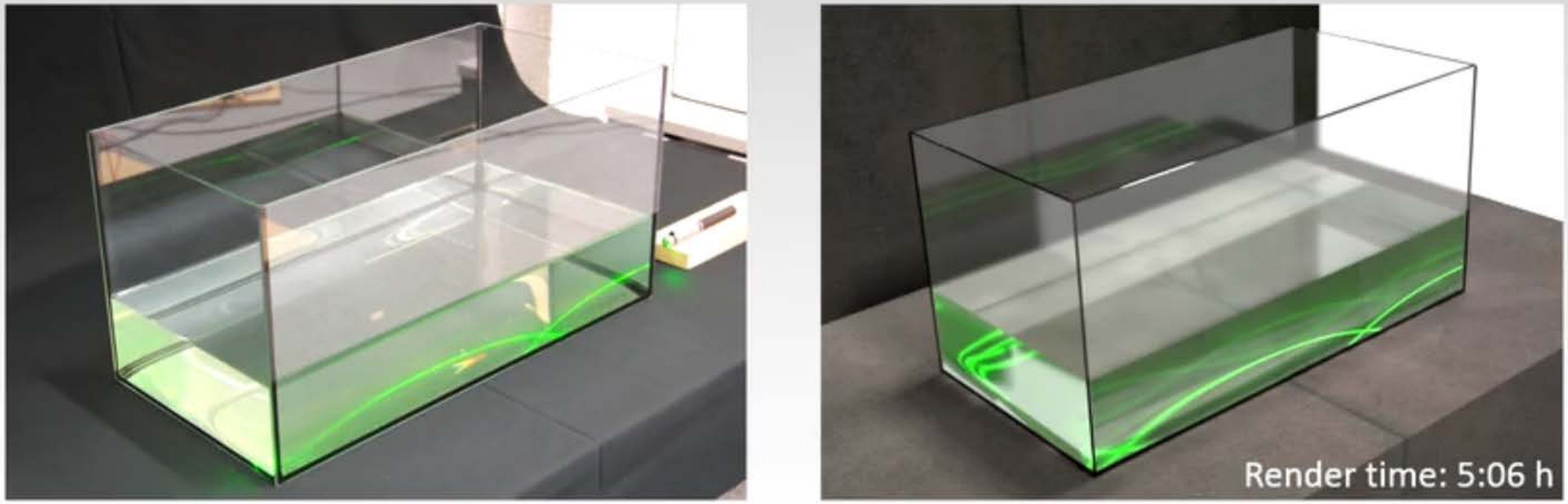
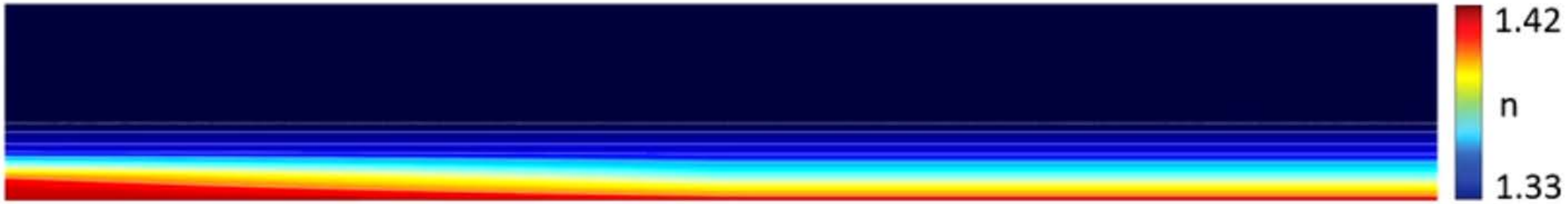
Experiments

Bending a Laser Beam



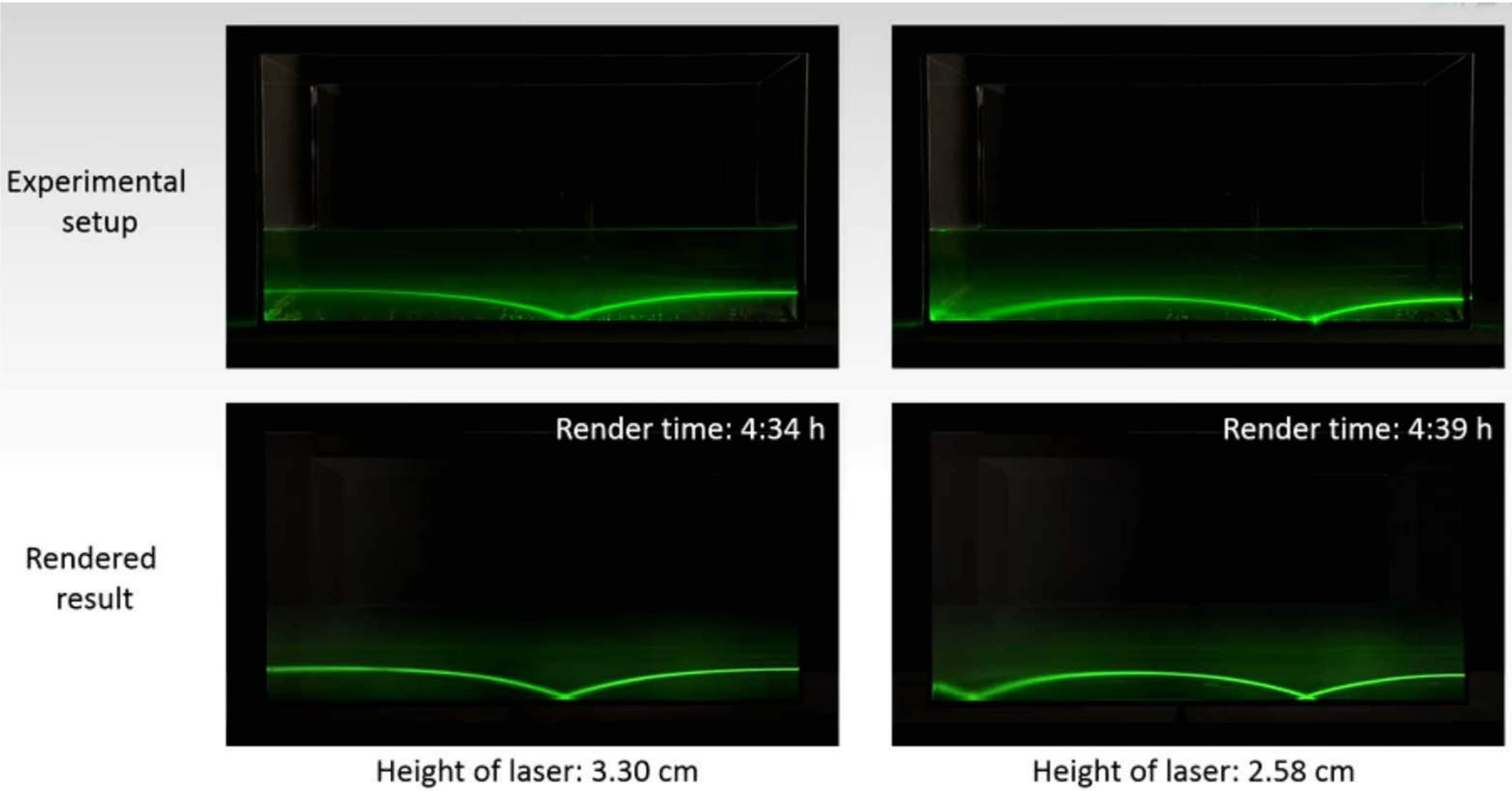
Experimental setup

Rendered result

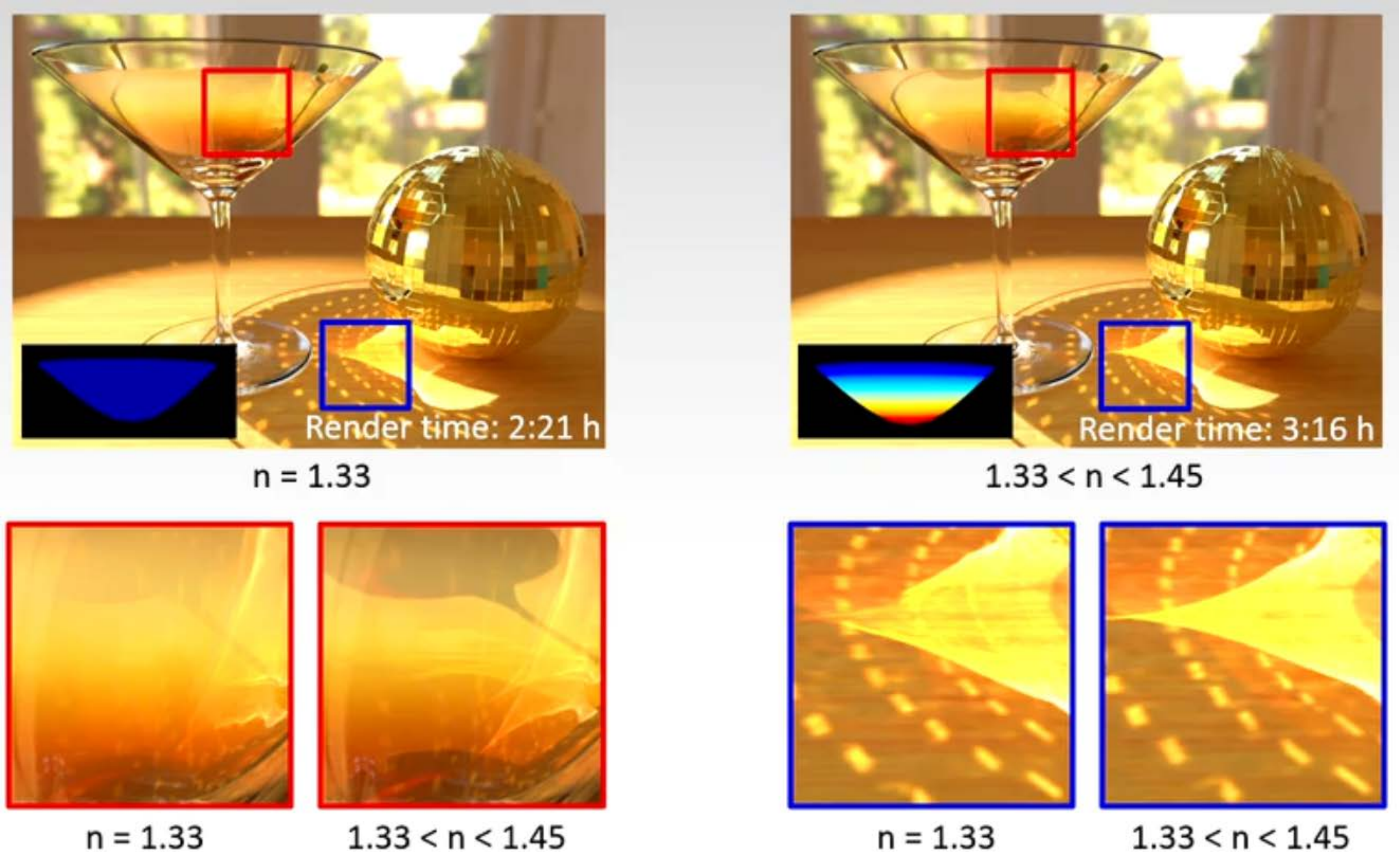


Index of refraction

Bouncing Laser Beams

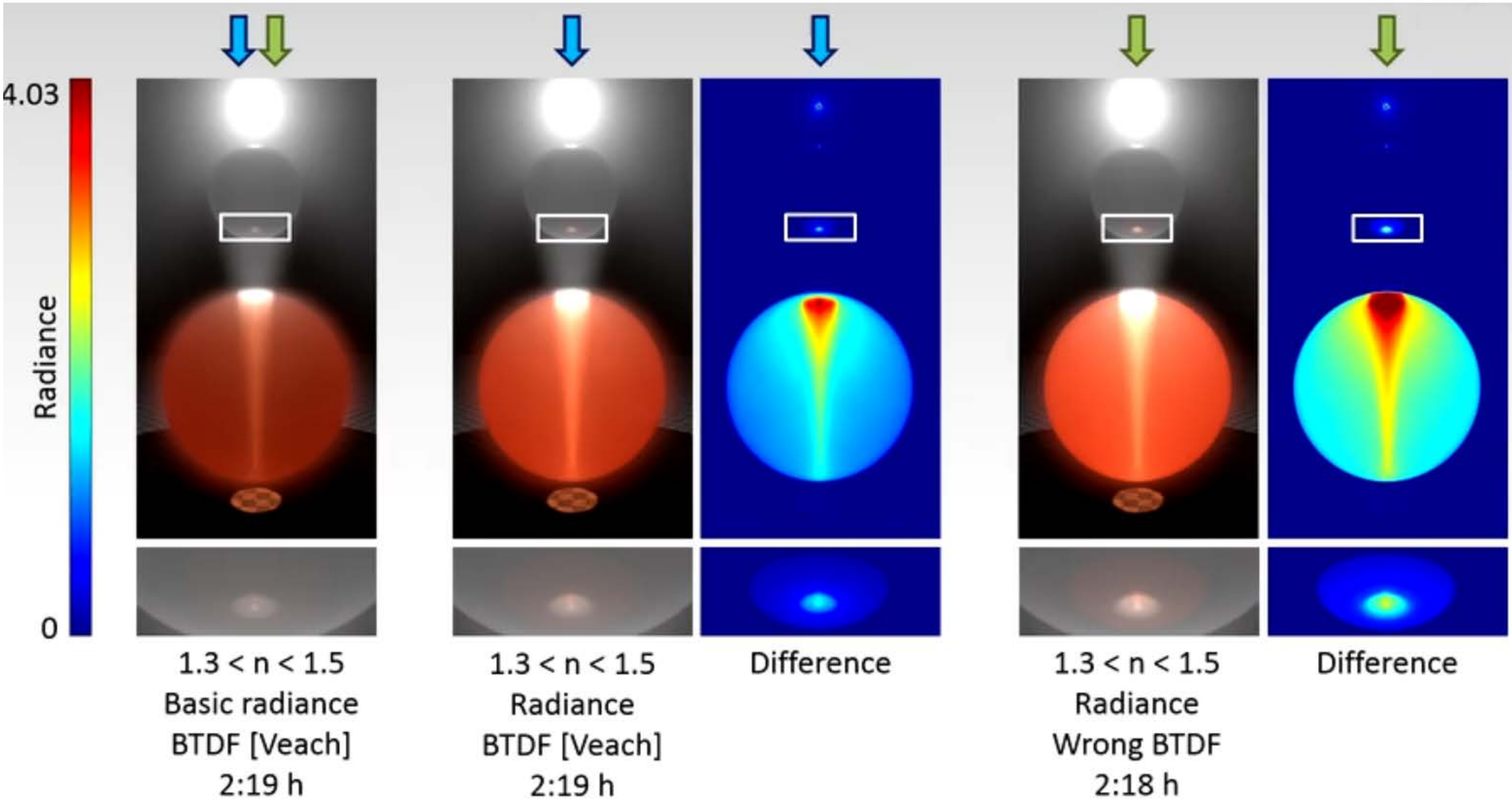


Cheers



Render time: 2:21 h
Render time: 3:16 h
n = 1.33
1.33 < n < 1.45
n = 1.33
1.33 < n < 1.45
n = 1.33
1.33 < n < 1.45

Conservation of Energy



Conclusion

Conclusion

- Light transport equation for continuous refraction
  - Strictly based on physics
  - Basic radiance as fundamental quantity
  - Complies with discontinuous function
- Involved theory, but simple to apply in practice
  - Continuously scale radiance with $1/n^2$
  - Visual impact is relevant for physically accurate results

Thank you for listening